

Równania Maxwella dla dielektryka $rot\vec{\mathcal{H}} - \dot{\vec{\mathcal{D}}} = 0$ $rot\vec{E} + \dot{\vec{\mathcal{B}}} = 0$ $div\vec{\mathcal{D}} = 0$ $div\vec{\mathcal{B}} = 0$ (1)

Równania materiałowe $\vec{\mathcal{D}} = \bar{\epsilon}\vec{E}$ $\bar{\epsilon} = \epsilon_0 \begin{pmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{pmatrix} = \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$ $\vec{\mathcal{B}} = \mu\vec{\mathcal{H}}$ (2)

Równanie fali płaskiej $\vec{E}(\vec{r}, t) = \vec{E} \exp[i(\omega t - \vec{k} \cdot \vec{r})]$, $\vec{\mathcal{H}}(r, t) = \vec{H} \exp[i(\omega t - \vec{k} \cdot \vec{r})]$ (3)

(1)+(2)+(3)
$$\begin{cases} n\hat{s} \times \vec{E} = \mu c \vec{H} \\ n\hat{s} \times \vec{H} = -c \bar{\epsilon} \vec{E} \end{cases}$$
 (4)

(4a)+(4b)
$$n^2 \hat{s} \times (\hat{s} \times \vec{E}) + c^2 \mu \bar{\epsilon} \vec{E} = 0$$
 (5)

Równanie charakterystyczne
$$\begin{bmatrix} n_x^2 - n^2(1 - s_x^2) & n^2 s_x s_y & n^2 s_x s_z \\ n^2 s_x s_y & n_y^2 - n^2(1 - s_y^2) & n^2 s_y s_z \\ n^2 s_x s_z & n^2 s_y s_z & n_z^2 - n^2(1 - s_z^2) \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$
 (6)

Rozwiązanie (6)
$$\det \begin{vmatrix} n_x^2 - n^2(1 - s_x^2) & n^2 s_x s_y & n^2 s_x s_z \\ n^2 s_x s_y & n_y^2 - n^2(1 - s_y^2) & n^2 s_y s_z \\ n^2 s_x s_z & n^2 s_y s_z & n_z^2 - n^2(1 - s_z^2) \end{vmatrix} = 0$$
 (7)

Wartości własne (6)
$$\boxed{\begin{aligned} n^4 - An^2 + B &= 0 \\ A &= \frac{s_x^2 \epsilon_x (\epsilon_y + \epsilon_z) + s_y^2 \epsilon_y (\epsilon_x + \epsilon_z) + s_z^2 \epsilon_z (\epsilon_x + \epsilon_y)}{s_x^2 \epsilon_x + s_y^2 \epsilon_y + s_z^2 \epsilon_z} \\ B &= \frac{\epsilon_x \epsilon_y \epsilon_z}{s_x^2 \epsilon_x + s_y^2 \epsilon_y + s_z^2 \epsilon_z} \end{aligned}}$$
 (8)

Wektory własne (6)
$$\vec{E} = \begin{bmatrix} \frac{s_x}{n^2 - n_x^2} \\ \frac{s_y}{n^2 - n_y^2} \\ \frac{s_z}{n^2 - n_z^2} \end{bmatrix} \quad \vec{\mathcal{D}} = \epsilon_0 \begin{bmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{bmatrix} \vec{E} = \epsilon_0 \begin{bmatrix} \frac{s_x n_x^2}{n^2 - n_x^2} \\ \frac{s_y n_y^2}{n^2 - n_y^2} \\ \frac{s_z n_z^2}{n^2 - n_z^2} \end{bmatrix}$$
 (9)